classmate

Date
Page

synthetic division

(Q1) Find the quotient and the remainder when $x^4 - 6x^3 + 10x^2 + 8x + 5$ is divided by $x - 3$.

Solution: →

3	1	-6	10	8		5
		3	-9	3		33
	1	-3	1	11		$38 = R.$

Thus the quotient is $x^3 - 3x^2 + x + 11$ and the remainder is 38.

(Q2) Find the quotient and the remainder when $x^5 + 2x^4 - 15x^3 + 12x + 10$ is divisible by $x - 4$.

Solution: →

4	1	2	-15	12	12	10
		4	24	36	144	624
	1	6	9	36	156	634

Thus

$$Q(x) = x^4 + 6x^3 + 9x^2 + 36x + 156 \text{ \&}$$

$$R(x) = \underline{634}.$$

classmate

Date
Page

(8) Find the HCF of the polynomials
$f(x) = x^4 + x^3 + 2x^2 + x + 1$ & $g(x) = x^3 - 1$
and express the HCF in the form
$t(x) f(x) + s(x) \cdot g(x)$.

Solution: →

$$
\begin{array}{r|l}
x^3-1 & x^4+x^3+2x^2+x+1 \; (x+1 \\
 & \underline{x^4 \qquad\qquad\quad - x} \\
 & \quad -\qquad\qquad\qquad + \\
 & x^3+2x^2+2x+1 \\
 & \underline{x^3 \qquad\qquad -1} \\
 & -\qquad\qquad\quad + \\
 & 2x^2+2x+2
\end{array}
$$

$$
2 \,\big|\, 2x^2+2x+2
$$

$$
\begin{array}{r|l}
x^2+x+1 & x^3-1 \; (x-1 \\
 & \underline{x^3+x^2+0x} \\
 & -\quad - \quad - \\
 & -x^2-x-1 \\
 & \underline{-x^2-x-1} \\
 & \times
\end{array}
$$

$\therefore$ HCF $= x^2 + x + 1$

Now, $f(x) = x^4 + x^3 + 2x^2 + x + 1$

$= (x^3-1)(x+1) + 2(x^2+x+1)$

$= g(x)(x+1) + 2(x^2+x+1)$

$\Rightarrow 2(x^2+x+1) = f(x) - (x+1)g(x)$

Hence H.C.F. $= (x^2+x+1) = \frac{1}{2} f(x) - \frac{(x+1)}{2} g(x)$

classmate

Date

Page

$$= t(x)\, f(x) + s(x)\, g(x)$$

Where $t(x) = \frac{1}{2}$ & $s(x) = -\frac{x}{2} - \frac{1}{2}$.

Remark:-

Common roots of two given equations, we should find their H.C.F.

(8) Find the common roots of the equations

$x^3 - 6x^2 + 11x - 6 = 0$ & $2x^4 - 7x^3 + x^2 + 7x - 3 = 0$

Ans:→

$x = 1, 3$

Depression of an equation when a relation exists between two roots.

$\alpha,\ \beta = \phi(\alpha)$

$f[\phi(\alpha)] = 0$ & ~~f&~~

→ α is a common root of the equations

$f(x) = 0$ & $f[\phi(x)] = 0$

& $\beta = \phi(\alpha)$.

M.U-94

Ex ① The equation $x^3-5x^2-4x+20=0$ has two roots whose difference is 3. Find all the roots of the equation.

Solution: →

Let α, β, γ be the roots of the equation

$$f(x) = x^3-5x^2-4x+20=0$$

Let us suppose that $\beta = \alpha+3$.

$\because$ β is a root of the equation $x^3-5x^2-4x+20=0$

$\therefore f(\beta)=0$ i,e; $f(\alpha+3)=0$

Hence $(\alpha+3)^3 - 5(\alpha+3)^2 - 4(\alpha+3)+20=0$

$$\Rightarrow [\alpha^3+3\alpha^2\cdot 3 + 3\alpha\cdot 3^2+3^3] - 5(\alpha^2+6\alpha+9) - 4(\alpha+3)+20=0$$

$$\Rightarrow (\alpha^3+9\alpha^2+27\alpha+27) - 5(\alpha^2+6\alpha+9) - 4(\alpha+3)+20=0$$

$$\Rightarrow \alpha^3+4\alpha^2-7\alpha-10=0 \quad \text{——— (1)}$$

Also α is a root of equation $f(x)=0$

$$\therefore \alpha^3-5\alpha^2-4\alpha+20=0 \quad \text{——— (2)}$$

Now we have to find out HCF between the L.H.S of (1) and (2). For this,

$$\alpha^3+4\alpha^2-7\alpha-10 \,\big)\, \alpha^3-5\alpha^2-4\alpha+20 \,\big(\, 1$$

$$\alpha^3+4\alpha^2-7\alpha-10$$

$$- \quad - \quad + \quad +$$

$$-3 \,\big|\, -9\alpha^2+3\alpha+30$$

$$3\alpha^2-\alpha-10$$

$$3\alpha^2-\alpha-10 \,\big)\, \alpha^3+4\alpha^2-7\alpha-10 \,\big(\, \alpha+13$$

$$3$$

$$3\alpha^3+12\alpha^2-21\alpha-30$$

$$3\alpha^3-\alpha^2-10\alpha$$

$$- \quad + \quad +$$

$$13\alpha^2-11\alpha-30$$

$$3$$

$$39\alpha^2-33\alpha-90$$

$$39\alpha^2-13\alpha-130$$

$$- \quad + \quad +$$

$$-20 \,\big|\, -20\alpha+40$$

$$\alpha-2$$

$$\alpha-2 \,\big)\, 3\alpha^2-\alpha-10 \,\big(\, 3\alpha+5$$

$$3\alpha^2-6\alpha$$

$$- \quad +$$

$$5\alpha-10$$

$$5\alpha-10$$

$$\times$$

Hence the HCF is $\alpha - 2$ which means that the common root of (1) & (2) is $\alpha = 2$.

$$\therefore \quad \beta = \alpha + 3 = 5$$

The third root can be ~~formed~~ found by dividing $f(x)$ by $(x-2)(x-5)$ and it is found to be $x = -2$.

2	1	-5	-4	20
		2	-6	-20
5	1	-3	-10	0
		5	10	
	1	2	0	

i.e, $x + 2 = 0 \Rightarrow x = -2$.

Therefor the roots of the given equation are, 2, 5 and -2.

Classmate
Date
Page

(Q1) One of the roots of the equation $x^3+x^2-x+15=0$ is -3. Find the other roots.

Solution:→

Since $x=-3$ is a root of the equation $x^3+x^2-x+15=0$.

$\therefore (x+3)$ must be a factor of $x^3+x^2-x+15=0$

-3	1	1	-1	15
		-3	6	15
	1	-2	5	0

That is,

$$x^3+x^2-x+15=(x+3)(x^2-2x+5)$$

Thus the other roots of the equation are obtained by solving $x^2-2x+5=0$

Now, from $x^2-2x+5=0$

$$x=\frac{2\pm\sqrt{4-4\cdot1\cdot5}}{2}$$

$$=\frac{2\pm4i}{2}=1\pm2i$$

Hence the other roots of the given equation are $1+2i$, $1-2i$.

Ex → Form the equation with rational coefficien which shall have for two of its roots $\sqrt{3}$ and $2+i$.

Solution:→ We know that in an equation, irrational roots and also imaginary roots occurs in pairs.

It follows ~~therefor~~ therefor that if $\sqrt{3}$ is a root of the given equation, the $-\sqrt{3}$ should also be a root; similarly if $2+i$ be a root of the given equation then its conjugate $2-i$ must also be its root.

Thus the required equation will be

$$(x-\sqrt{3})(x+\sqrt{3})\{x-(2+i)\}\{x-(2-i)\}=0$$

$$\Rightarrow (x^2-3)\{(x-2)-i\}\{(x-2)+i\}=0$$

$$\Rightarrow (x^2-3)\{(x-2)^2+1\}=0$$

$$\Rightarrow (x^2-3)(x^2-4x+5)=0$$

$$\Rightarrow x^4-4x^3+2x^2+12x-15=0$$

Q) Form a rational equation which shall have for two of its root $1+i$ & $2-3i$

Ans:- $x^4-6x^3+23x^2-34x+26=0$

classmate

Date

Page

Ex-(6) If $\alpha_1, \alpha_2, \cdots \alpha_n$ be the roots of the equation

$$(\beta_1 - x)(\beta_2 - x) \cdots (\beta_n - x) + A = 0$$

find the equation whose roots are $\beta_1, \beta_2, \cdots, \beta_n$.

Solution:-

Since $\alpha_1, \alpha_2, \cdots, \alpha_n$ are the roots of the given equation, therefor we have

$$(\beta_1 - x)(\beta_2 - x) \cdots (\beta_n - x) + A \equiv (\alpha_1 - x)(\alpha_2 - x) \cdots (\alpha_n - x)$$

$$\Rightarrow (\beta_1 - x)(\beta_2 - x) \cdots (\beta_n - x) \equiv (\alpha_1 - x)(\alpha_2 - x) \cdots (\alpha_n - x) - A$$

This shows that $\beta_1, \beta_2, \cdots, \beta_n$ are the roots of the equation

$$(\alpha_1 - x)(\alpha_2 - x) \cdots (\alpha_n - x) - A = 0.$$